Comment on "Role of Initial Entanglement and Non-Gaussianity in the Docoherence of Photon-Number Entangled States Evolving in a Noisy Channel"

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In [1], Allegra *et al.* employ several entanglement criteria, including the Simon criterion (SI), in order to provide evidence to support their conjecture that a Gaussian state remains entangled longer than a non-Gaussian state in a noisy Gaussian channel. In particular, they study the loss of entanglement for the class of photon number entangled states (PNES) $|\Psi\rangle = \sum_n \Psi_n |n\rangle|n\rangle$ in a thermal reservoir. Here we show that their evidence is seriously flawed due to their use of entanglement criteria inappropriate for the comparison and that there exist a large class of non-Gaussian entangled states even within the PNES that can be more robust than Gaussian states.

The dynamics of a system under two independent Markovian reservoirs can be described by $\dot{\rho} = A\sum_{i=1,2}\mathcal{L}[a_i]\rho + B\sum_{i=1,2}\mathcal{L}[a_i^\dagger]\rho$, where A and B denote the interaction strength leading to dissipation and amplification, respectively ($\mathcal{L}[O]\rho \equiv 2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O$). The case of $A > B$ describes the interaction with a thermal reservoir. Using the notations in [1], $A = \frac{\Gamma}{2}(N_T + 1)$ and $B = \frac{\Gamma}{2}N_T$ (N_T: thermal photon number, Γ: decay rate). Allegra *et al.* found that the Simon criterion (SI) based on the symplectic eigenvalues under partial transposition (PT) is optimal for PNES among the criteria they considered. Here, in contrast to [1], we employ the negativity of the density matrix under PT (NDPT) as the entanglement criterion and compare the entanglement dynamics of two particular PNES states studied in [1]: photon subtracted squeezed vacuum (PSSV : non-Gaussian) and the twin-beam state (TWB: Gaussian). For numerical purposes, we checked the negativity by restricting the elements of the whole density matrix to a subspace truncated by dimension $N_{\rm tr}$. It is well-known that the negativity under PT in a subspace is sufficient to verify entanglement. In Fig. 1 (a), we show the time to lose negativity for PSSV by NDPT ($N_{\rm tr} = 3$) together with the separation time of TWB by SI (analytic result). These results clearly indicate that the PSSV remains entangled longer than the TWB for either the same initial entanglement ϵ_0 or the same energy (indiscernible in this case).

The failure of Allegra *et al.* is attributed to the wrong choice of entanglement criteria; Markovian interaction would not create a new-type of correlation, thus, an initial entanglement witness will remain a significant tool to verify the entanglement of the decohered state at a later time. For a given state, it is usually nontrivial to identify the most efficient entanglement witness, however, it is obvious that the information on non-Gaussian entanglement is not fully contained in the covariance matrix (SI criterion). We instead checked the negativity of the decohered PNES under PT, which is a well-known tool particularly for continuous variables. The NDPT can show the robustness of entanglement for a non-Gaussian PNES, even analytically. In [1], they also studied randomly-generated PNES in a truncated basis, only to confirm their incorrect conclusion. Among these, the simplest one is $|\Phi\rangle_1 = c_0|00\rangle + c_1|11\rangle$, for which the entanglement can be initially detected by the negativity in the subspace spanned by $|01\rangle$ and $|10\rangle$ under PT, which will be presumably useful also at later times. The separation time in this subspace can be analytically obtained by direct calculation. Fig. 1 (b) shows the value of B/A above which $|\Phi\rangle_1$ survives longer than the TWB with the same initial energy (blue) or entanglement (purple) as a function of $|c_1|^2$, clearly demonstrating the failure of the evidence of Allegra *et al.* in a wide range of temperatures.

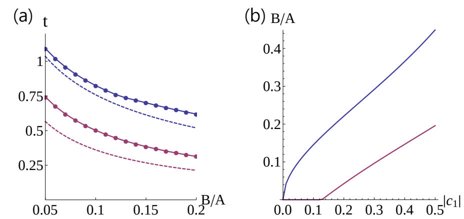

FIG. 1. (a) Time to lose the negativity in PT of PSSV (solid with circles) and the separation time of TWB (dotted) with $\epsilon_0 = 0.1$ (purple) and $\epsilon_0 = 1$ (blue) (b) the value of B/A above which $c_0|00\rangle + c_1|11\rangle$ survives longer than TWB.

In summary, we have shown that the evidence of Allegra *et al.* supporting their conjecture is derived from the wrong choice of entanglement criteria and disproved the maximal robustness of Gaussian entangled states.

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